Properties of Proton in diquark Model

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Abstract: The properties of the proton has been investigated considering a proton as diquark-quark system. A model for diquark has been suggested in an analogy with the quasi-particle in a crystal lattice. The mass of the diquark obtained in the present model has been found to be in good agreement with other theoretical predictions. The binding energy of proton, compressibility and excitation energy for the Roper Resonance have also been estimated in the present model and are found to be in agreement with the existing theoretical and experimental findings.

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Diquarks play an important role in the structure of hadrons. It is now growing idea that the deeply bound diquarks are the building blocks in the formation of the meson and baryon states and the exotics. Some experimental facts like hadron jet formation etc demands the prediction of diquark and the role of diquark in baryon spectroscopy, deep-inelastic function and dynamics has been discussed in details by Ida et al.and Anselmino et al [1]. Particularly after the discovery of the pentaquark baryon θ^+ [2], the search on diquark attract much attention. A number works have been done towards the understanding of the structure of diquark. The possibility of forming quark-quark and quark-antiquark system by Instanton Induced Interaction(III) have been developed by Shuryak [3] and Schafer et.al [4]. Betman et.al. have investigated formation of bound state of quark-quark or quark-antiquark systems due to instanton induced interaction and predicted such bound state in the hadron as a bubble of the size of the instanton radius. In QCD both gluon exchange interaction or instanton induced interaction favour spin singlet colour- antisymmetric diquark combination. The discovery of the exotic baryon θ^+ which has the mass much less than the constituent quark, the diquark picture in the framework of III model reproduces much satisfactory result. A number of works have been done on the diquark picture of the pentaquark [5]. If diquark truely exists and proved to be a fundamental building block of hadrons, it must also reproduce the nucleon or other baryon properties as well. In the present work we have suggested an alternative picture of diquark in which two quarks bound together forming a quasi particle in analogy with the quasi particle formed in the crystal lattice [6]. We have estimated the diquark mass in the framework of the quasi-particle in the lattice and the results are obtained is in good agreement with the other theoretical predictions. We have estimated the proton binding energy, compressibility and excitation energy of the Roper resonance,the first excited state of the nucleon in this quasi-particle model and obtained favourable results.

The electron in crystal is in a situation exactly the same as the elementary particle in vacuum [7]. The force will be acting on particle which are already under crystal field. An crystal electron is subjected to two force namely the effect of crystal field (∇V) and an external force (F) which accelerate the electron. So the electron in a crystal behaves like a quasi particle whose effective mass m^* reflects the inertia of electrons which are already in a crystal field such that:

$$m^* \frac{dV}{dt} = F_h \tag{1}$$

and the bare electrons (with normal mass) are affected by the lattice force -∇V, which corresponds the periodic crystal potential V as well as the external force F. so that:

$$m \frac{dV}{dt} = F - \frac{dV}{dx} \tag{2}$$

Hence the ratio of the normal mass (m) to the effective mass (m^*) can expressed as:

$$m/m^* = 1 - \frac{1}{F}[\frac{\delta \overline{V}}{\delta x}] \tag{3}$$

We propose a similar type of picture for diquark as quasi particle inside a nucleon. To get diquark effective mass inside the nucleon we assume that the diquark is an independent body which is under the influence of one gluon exchange type of field due to the meson cloud represented by potential $\overline{V}$ =-(4/3)α_s/r in analogy with the crystal field on a crystal electron. and an average of lattice force F = -ar as an oscillatory external force so that the the ratio of the constituent mass and the the effective mass of the diquark inside a hadron may be represented as:

$$\frac{m}{m^*} = 1 + \frac{\alpha_s}{ar^3} \tag{4}$$

Here m represents the normal constituent mass of the diquark and m*is the effective mass of the diquark,$\overline{V}$ being the average value of the one gluon exchange potential. To calculate the effective mass of the diquark we need the parameter r for the diquark. In the context of discussing the diquarks in instanton induced interaction model Betman et.al. [5] have considered the instanton spacial distribution function F(r) as F(r) =δ(r-ρ_c where ρ_c is the characteristic instanton size which also represents radius of the bound state of diquark in the form of bubble of the size of the instanton radius. We have assumed that the'r' parameter for the diquark is of the order of ρ_c and taken to be = 0.4 fm as in [5]. With α_s = .59 [8] for light hadrons and r = 0.4 fm, m =.72GeV (m_u= .36GeV) and a = .06 [9], we get the effective mass of the diquark as 272 MeV in the quasi particle model of the diquark. Jaffe et al. [10] estimated the diquark mass 420 MeV in the context of III interaction whereas Kl Model [11] estimated the mass to 209 MeV. Some other calculations [5,12] estimated the mass in the range between 600 to 800 MeV.

To estimate the binding energy of proton we consider ud-u picture of proton where (ud_0) constitutes the diquark of mass 272 MeV. We then estimated the reduced mass of the diquark-quark (M_R) system and consider that a particle of mass M_R is moving under a potential V(r) = ar^2 with the centre of mass fixed to the centre of the baryonic sphere. The reduced mass is estimated to be as 0.155 GeV. It may be mentioned here that Krolikwoski [13] has made one body equilateral triangle approximation to the three(Dirac) particle system for estimation of the Hamiltonian. Here the expression for the Hamiltonian corresponding to the particle of mass M_R moving in a background potential V(r)= ar^2 due to the sea contribution runs as:

$$H = -\frac{\hbar^2}{2M_R} + ar^2 \tag{5}$$

The expectation value of H corresponding to the wave function $\psi(r)$ is;

$$< H >= \int \psi^* H \psi d^3r \tag{6}$$

The lowest upper bound of the ground state energy for a wave function ψ may be recast as:

$$E_0 \leq \int \psi^* H \psi d^3r \tag{7}$$

In the context of the Statistical model [14] the square of the wave function of a proton for harmonic oscillator type of background potential is obtained as:

$$\psi(r) = A^{\frac{1}{2}}(r_0^2 - r^2)^{\frac{3}{4}} e^{i\alpha}\theta(r_0 - r) \tag{8}$$

Where r_0 corresponds to the radius parameter of a proton. θ is usual step function.α is a constant phase factor, A = $\frac{8r_0^{-6}}{\pi^2}$[13] From the above expression we have the —$\nabla\psi$— and corresponding $< T >$ has been estimated with the above baryonic wave function as:

$$< T >= 43.49 r_0^{-2} \tag{9}$$

The expectation value of the potential energy has been estimated as:

$$< V >= 0.375 a r_0^2 \tag{10}$$

Hence the total energy runs as:

$$< E >= 43.49 r_0^{-2} + 0.375 a r_0^2 \tag{11}$$

r_0 is typical radius parameter of the proton and can be estimated using the experimental data corresponding to form factor. The form factor of proton has been obtained from the expression $F(q^2) = \int e^{iq.r} |\psi(r)|^2 d^3r$. With the above $|\psi(r)|^2$ as input we obtain for $q^2 \longrightarrow 0$;

$$F(q^2) = 1 - 0.057 r_0^2 q^2 \tag{12}$$

The relation between the proton charge radius and form factor for a proton is:

$$F(q^2) = 1 - \frac{1}{6} < r_{ch}^2 > q^2 \tag{13}$$

Comparing above expressions for $F(q^2)$ we obtain; $< r_{ch}^2 > = 0.6 r_0$. The proton charge radius is obtained as [15] $< r_{ch}^2 >^{\frac{1}{2}} = 0.88$fm which yields r_0= 1.46fm = 7.33GeV. With the input of this radius we obtain $< E > = 1.927$ GeV. Lim [16] has solved the three body (trinucleon) problem with the spin dependent internuclear harmonic oscillator potential and obtained the binding energy of the triton as:

$$E = (\frac{2\hbar^2}{m})^{1/2} \{ (3\omega V_1 + \frac{3}{2} b_1 V_1)^{1/2}) + (3\omega V_1 - \frac{3}{2} b_1 V_1)^{1/2} \} - 3\omega V_0 \tag{14}$$

where V_0 V_1, b_1, ω are appropriate constants. Considering the three body system as the three quark system with mass of m as m_q (360 MeV),quark mass, the binding energy for ground state of proton is obtained as $< E > = 1.997$ GeV (with $\frac{1}{2} K r^2 = V_2 r^2 = V_1 \omega r^2 =$ ar^2. So we find that the our estimate of the binding energy for proton in the context of the quasi-particle model agree closely with the Lim's exact theoretical calculation of the ground state energy.

It has been pointed by Morsch et al. [17] that the information on the compressibility of a system can be obtained from the dynamical properties of the size degree of freedom in radial mode.The compressibility of a nucleon is given by the expression;

$$K = r_0^2 \frac{1}{3}(d^2E/dr^2) \tag{15}$$

To get an estimate of the compressibility of nucleon we use the radius parameter of nucleon = 7.33GeV as above and get the value as K_N = 1.61GeV The Roper excitation energy is given by:

$$\Delta E = \frac{K_N}{m_q r_0^2} \tag{16}$$

We estimated the Roper excitation energy as 288 MeV in the present work. However it may be mentioned that the Roper resonance form factor is smaller than proton form factor which indicates the fact that the Roper being a more diffuse system than proton[18]. Morsch et al.[17] have investigated $P_{11}(1440)$ in the alpha-photon scattering. With the mean quare radius equals to $0.62fm^2$, they have extracted the $K_N = 1.4 \pm 0.3$GeV. MIT [19] Bag model predicts the value in between 900 to 1200 Mev whereas the constituent quark model yields the value as $\simeq$ 3Gev. Mathieu et el [20] have estimated the value as 636 MeV in the context of the flux tube model. Hoodbhoy et.al [21] have investigated the pion mediated interaction in a chiral bag model considering nucleon size and compressibility as a parameter. They have taken K_N as 2GeV and 3GeV with radius parameter as .6fm to.8fm. Meissner et.al.[22] estimated the compressibility as 4GeV.

We have estimated the excitation energy of Roper resonance as 288 MeV with K_N = 1.61 GeV as estimated by us. Meissner et al.[22] have estimated the excitation energy to be 390 Mev. However they have mentioned that all the relativistic approaches estimate the roper resonance excitation energy ranging from 200MeV to 500MeV whereas the experiment predicts the value as 500Mev.

In the present work we have suggested a quasi particle picture of diquark which resembles a quasi particle in a crystal lattice. It may be pertinent to mentioned here that in the context of investigating the super conducting properties of a hadrons [23], it has been suggested that vacuum is supposed to contain a sea of virtual q$\overline{q}$ pairs condensates somewhat similar to the situation of Cooper pairs in a superconductor. The particle picture description of hadrons may have some relevance to the recently developed idea that diquarks are the building blocks of hadrons and exotics. The binding energy we obtain does not mean the binding energy in usual sense as quarks are not free but binding energy should be more than the mass of the proton. We have observed that if a trinucleon system is replaced by three quarks to represent a proton the binding energy obtained from the exact solution of Lim [16] is almost equals to our calculation obtained from the quasi particle picture of diquark-quark of proton. The compressibility of the nucleon estimated in the present work lies in the range of recent extracted value [17]. The Roper excitation energy is obtained in the range of other theoretical estimates. It should be mentioned that most uncertainty lies in the estimation of the radius parameter which is not very well known [17]. In the context of discussing the pentaquark baryons Oka [6] has pointed out that situation is not very clear with the diquark picture but diquarks if exists should be reexamined with the other ground state baryons. The model we have suggested for diquaks reproduces the properties of proton in existing theoretical and experimental limits and may not be far from reality. However further investigations would be made with the exotics particularly with the pentaquark system in our future works.

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